Type A and Type B Effects, Time-Order Error and Weber's Law in Human Timing – Simulations and Synthesis

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Abstract

This article presents a computational approach to the theoretical integration of the psychophysical phenomena in human timing. While there are many useful models of human timing, analyses are scarce on how these models explain the relationships between several phenomena at the same time. The presented research is an attempt to primarily explain and integrate the time-order error with the Type A and Type B phenomena. The final result of this work also encompasses Weber's law property and relates it to the aforementioned order-related effects. The theoretical framework used is the Clock-Counter Timing Network (CCTN), an artificial neural network timing model which has been constructed to explain the process of comparing durations of stimuli. Extensive simulations performed with the use of this model revealed that the considered psychophysical properties may be strongly interrelated and dependent on a simple perceptual mechanism. The obtained results allow to formulate specific experimentally testable predictions.

Contents

1 Introduction

Timing is one of the most important human and animal perceptual and cognitive processes. Properly functioning timing mechanisms are essential on many levels of perception ranging from sensory-level integration processes to higher-level processes, such as movement control or planning. This is why timing research covers so many domains such as behavioral studies on people and animals [54, 45, 75, 18], psychological experiments [79, 22], neuropharmacological experiments and neuroimaging experimental studies [25, 15, 65, 68, 64, 47, 23]. Among the most interesting research topics are inborn characteristics of timing mechanism and their possible merits and limitations.

One of the best ways to obtain and organize knowledge about timing is to construct models of the mechanism, analyze their properties and compare these models with data collected in real life experiments. The literature on timing offers many kinds of such models which can be divided into several classes. These classes often reflect domains in which timing is researched, but there are also more general models that try to integrate knowledge from various kinds of experiments. To enumerate the most important classes: there are psychophysical models represented in the form of equations grasping the psychophysical properties of timing [59, 30, 12, 8, 36, 16, 17], clock-counter models represented in the form of more or less complex architectures depending on the internal clock-mechanism – these models are often also based on psychophysical equations that enable to match the properties of the clock mechanism with patterns observed in the data [20, 19, 71, 77, 76, 78, 13], neural models [5, 67, 7, 35], and also more complex architectures [49, 9, 51]; more information about models is available in [1, 6, 31, 21, 81, 66, 80, 69]. The model that will be presented in this paper – the Clock-Counter Timing Network (CCTN) – is a hybrid model between clock-counter and neural models [39, 40, 41].

Timing literature is rich in descriptions of many effects related to timing, offering a great insight into the organization of human timing mechanisms (however see [50] for a review of problems that timing research is currently facing). Among such effects are Weber-law-like properties [74, 75, 14, 38], influence of stimuli sensory characteristics on the accuracy of timing [79, 57], and the time-order error. Time-order-related effects and Weber-law-like properties are the main focus of this paper.

1.1 The time-order error

The time-order error (TOE) is an effect revealed in psychophysical studies concerning comparison of magnitude of two subsequently presented stimuli [14, 27, 29, 25, 32, 2, 33, 13]. Many different sensory domains such as weight, loudness, pitch or even pain have been confirmed to possess the property of the time-order error. The domain of duration perception is no exception. Simply speaking, in the domain of timing the TOE means the over- or under-estimation of the duration of the first presented stimulus relatively to the second. It is often expressed as (half of) the difference between the frequency of the correct answer when the first stimuli lasted longer and the frequency of the correct answer when the second stimuli lasted longer. When the presented pair consists of stimuli of equal duration, the TOE is calculated as the difference between the answer “first stimuli lasted longer” and 50%. Formally, these two measures can be expressed as follows:

$$\mathrm{TOE} = \frac{\mathrm{P(CORRECT|FirstLonger)} - \mathrm{P(CORRECT|SecondLonger)}}{2} \tag{1}$$

$$\mathrm{TOE} = \mathrm{P(FirstLonger|BothIdentical)} - 0.5 \tag{2}$$

Hence, a positive value of the TOE means that the duration of the first stimulus was overestimated, and a negative value of the TOE means that the duration of the second stimulus was overestimated. Note that the TOE equal to 0 does not necessarily mean that a subject is 100% accurate – consider P(CORRECT|FirstLonger) = P(CORRECT|SecondLonger) = 0.5; see [41] for more extensive analyses of the relationship between the accuracy of answers and the TOE.

As research on the TOE has traditionally intertwined with the development of the psychophysics of timing, a number of factors influencing the magnitude and polarity of the TOE have been identified and studied [28, 2, 32]. Among those factors, the most basic ones are the range of duration of stimuli (milliseconds vs. seconds) and the duration of the time gap between stimuli called ISI (inter-stimulus interval).

1.2 The important distinction between the Type A and B phenomena and the time-order error

The literature introduces yet another concept of the time-order error which is derived from the analyses of psychometric functions in the studies where a fixed standard stimulus and a variable comparison stimulus are compared [14, 27, 12, 4, 73]. In this approach, the time-order error is calculated as follows:

$$\mathrm{TOE} = \mathrm{PSE}_{(\mathrm{S,C})} - \mathrm{PSE}_{(\mathrm{C,S})} \tag{3}$$

where PSE is the point of subjective equality (the value of the comparison stimulus judged to be longer than the standard stimulus with the 50% frequency), the (X,Y) pairs represent the order of presentation (where X is presented first), and S and C denote the standard stimulus and the comparison stimulus, respectively. As it is explained for example by [12]:

> A negative time-order error means underestimation of the first relative to the second stimulus. For stimulus order (S,C), underestimating the first stimulus (i.e., S) is equivalent to overestimating the second stimulus (i.e., C), that is, a smaller magnitude for C suffices in order to yield subjective equality to S, hence PSE<S. Conversely, for stimulus order (C,S), underestimating the first stimulus (i.e., C) results in PSE>S.

Due to this account differing much from the previously defined measure of the TOE, the difference between the PSE for the two orders of presentation was called the *Type A* effect (Type A, for short). The other reason for the name is that studies show that apart from the Type A effect, there is the *Type B* phenomenon (Type B, for short), also called the standard-position effect or the constant-position effect [30]. Type B is measured as follows:

$$\mathrm{Type}_{\mathrm{B}} = \mathrm{DL}_{(\mathrm{S,C})} - \mathrm{DL}_{(\mathrm{C,S})} \tag{4}$$

where DL is the difference limen. The difference limen is usually calculated as a halved difference between comparison stimuli values corresponding to 75% and 25% frequencies of the answers stating that the comparison stimulus is longer than the standard. DL helps to compare the slope of the psychometric functions and represents the sensitivity of the judgments made by participants; it is also used to represent the just-noticeable difference (JND). Studies confirmed the Type B effect [12, 11, 30, 4], and Dyjas and Ulrich even noted that it is more robust than the Type A phenomenon. In their research concerning the development of the Internal Reference Model (IRM), they have discovered two interesting facts concerning the relation between the Type A and the Type B effects. Firstly, in comparison studies the negative Type B effect may occur when the Type A effect is not present. Secondly, the Type B effect, contrary to Type A, is prevalent in various experimental tasks: the negative Type B effect was present both in the comparison task and the equality detection task, while the Type A effect (positive) was only present in the equality detection task.

Hence, an interesting question arises: what is the relation between the TOE and the Type A effect or the Type B effect? The relation between Type A and Type B themselves appeared to be unclear to the point that [12] stated (text in square brackets added for clarity):

> Taken together, these studies suggest that the Type B effect and shifts of the psychometric functions [the Type A effect] are at best loosely linked phenomena, rather than two sides of the same coin. In this respect, the Type A effect does not need to be in the scope of the IRM, which was developed to account for effects of stimulus order on discrimination sensitivity.

In this paper we will show by means of simulation experiments that our Clock-Counter Neural Network model (CCTN) integrates and explains all the three effects as originating from one source, and being in fact different sides of the same phenomenon. The explanation that will be provided is not, however, complete. This is because the CCTN is not adjusted to explain equality-judgment data – it was originally designed to explain comparative data only. This does not invalidate the perception-based or memory-based explanations of the mechanism of the TOE, but additionally indicates that further (possibly, decision) stages of the processing of temporal data may be responsible for the difference between the results of comparative and equality tasks. This hypothesis is strengthened by the observation that in the article by [12], while the negative Type B value was preserved in the equality-judgment task, the size of the effect was much lower (despite the fact that the used stimuli were the same as in the comparative task).

1.3 The CCTN and the TOE

The Clock Counter Timing Network introduced by [39, 40] is an artificial neural network model adopting many of the ideas of traditional clock counter models, mainly from one of the most popular models – the Scalar Timing Model (STM) proposed by Gibbon et al. [20, 77, 76]. Apart from being a neural implementation which enables relatively easy testing of the properties of the model in simulation, the CCTN extends the STM with additional mechanisms that produce the TOE in comparative judgments. The detailed analyses of the TOE in the CCTN are given in [41] and are extended in this paper.

The CCTN, as presented in Fig. 1, consists of several neural modules. Each module is built from one or more specialized neurons. Each neuron has zero or more inputs (left side of the neuron symbol) and one output (right side of the symbol). Neurons are simulated synchronously and the signal is a real value passed from an output to an input. Since detailed descriptions of the CCTN were provided earlier [41], here we do not duplicate previously published work and only present general assumptions concerning the model and focus on the explanation of the mechanisms behind the TOE. The central modules of the CCTN are: the Receptor, the Pacemaker, the Accumulator, the Reference Memory, the Working Memory, and the Comparator.

Let us start with a general description of the comparison process. The Pacemaker is a one-neuron module producing pulses according to the Poisson distribution. When the first stimulus in the trial appears (i.e., it is detected by the Receptor), the Accumulator starts to collect the pulses. After the end of the first stimulus, the value representing the duration of the first stimulus is sent from the Accumulator through the Scalar Variance

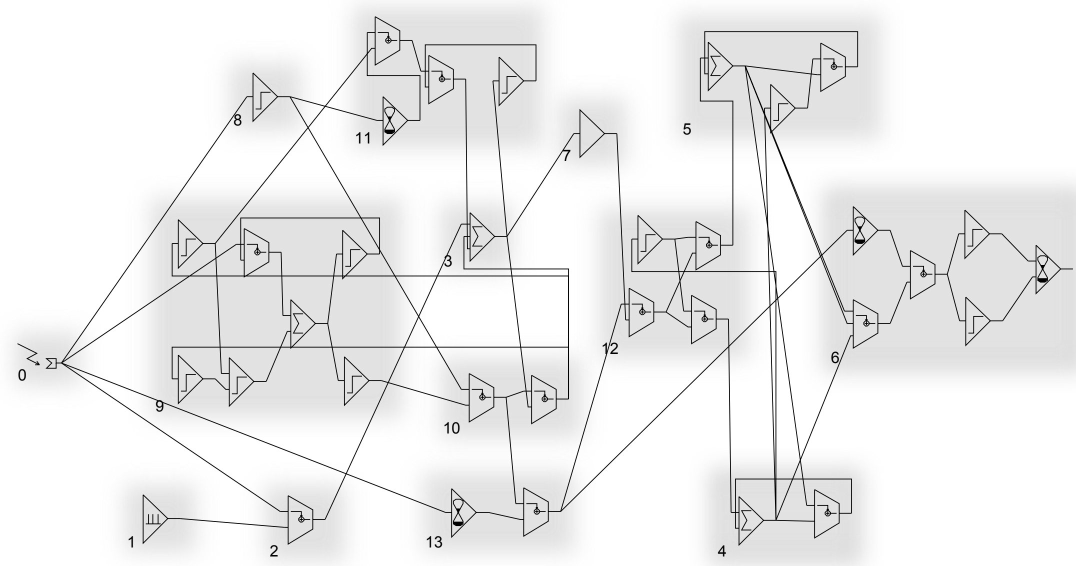

Figure 1: The diagram of the Clock Counter Timing Network. Selected artificial neuron modules shown as shaded boxes (0 – Receptor, 1 – Pacemaker, 3 – Accumulator, 4 – Reference Memory, 5 – Comparator, 6 – Scalar Variance Module) are briefly described in the text. For a more complete description of the function of the modules and the meaning of the symbols depicting neuron types, refer to [41].

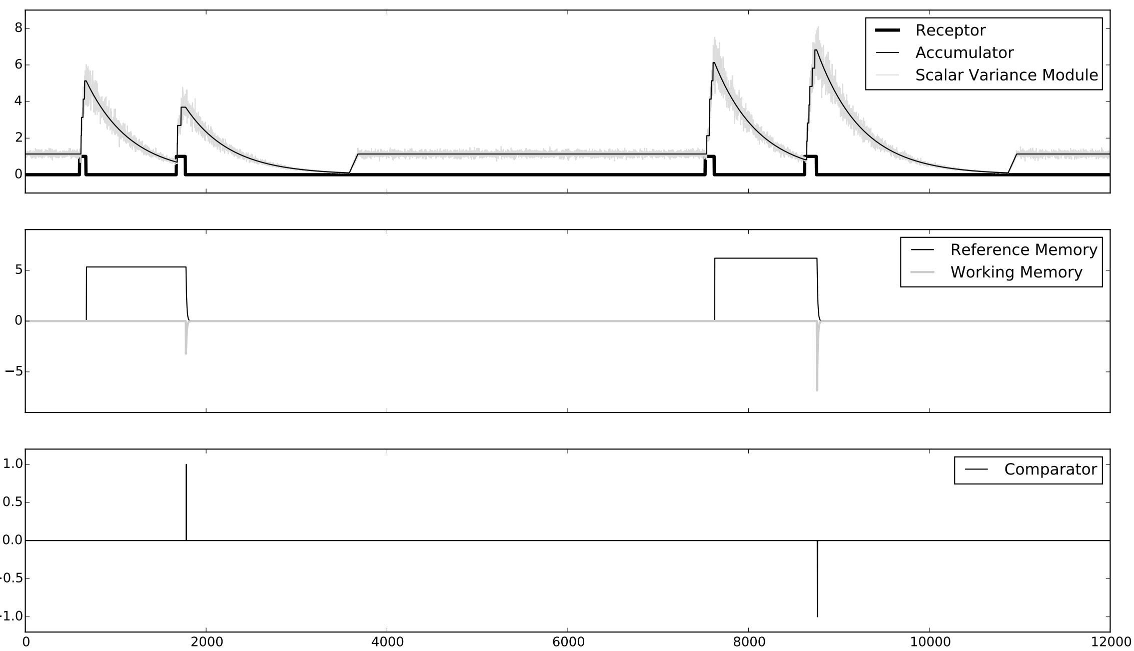

Figure 2: The signal waveforms output by the described modules of the CCTN model with horizontal axis representing time measured in simulation steps (abbreviated as 'ss' further in the text). Each plot shows changes in the signal output by these modules during the presentation of two pairs of stimuli. Values produced by the Comparator after the presentation of each pair of stimuli (i.e., the two peaks visible in the bottom plot) are stored during the simulation and used for further analyses reported in this paper.

Module to the Reference Memory module, and the Accumulator stops collecting pulses and it is cleared. The situation is repeated after the beginning of the second stimulus, however, after the end of the second stimulus, the information about the number of pulses is sent to the Working Memory. Next, the signals from the Working Memory and the Reference Memory are compared – if the first is greater than the second, the response "first longer" represented by signal 1.0 is output, otherwise −1.0 is output. The illustration of this process is shown in Fig. 2

Now, the TOE (not the Type A effect) in the CCTN emerges because of two mechanisms. First, the Accumulator, after its resetting process completes, is charged by the bias module. This bias is just some constant value (a scalar). The positive TOE is likely to appear when both stimuli are of short duration and the ISI is short: the number of pulses collected after the exposition of the first stimulus adds up to the bias value, and if the second stimulus starts when the Accumulator is cleared to the point below the bias value, then the number of pulses is added to the lower value than generated by the first stimulus. Under such conditions, the second stimulus is more prone to be underestimated. Actually, any duration length of stimuli can lead to a positive TOE provided that the ISI is carefully adjusted, however, for longer stimuli, the negative TOE is more likely to be observed. This is also more probable when additionally the ISI is relatively short. This is because if a large amount of pulses is stored in the Accumulator, then the Accumulator may not become cleared below the bias level on time. In this case, the pulses that represent the duration of the second stimulus will add up to the remnants of the first stimulus, thus increasing the chance of the second stimulus to be judged as longer.

According to the CCTN assumptions, it is not the absolute duration of stimuli or the ISI alone that determines the magnitude and the polarity of the TOE. It is the interplay of these two factors that is important. It may be that short stimuli with a really short ISI will produce a negative TOE, and longer stimuli with the appropriately selected ISI will produce a positive TOE. However, as the bias in the Accumulator is constant and it is usually much lower than the number of pulses collected even during short stimuli, and the rate of the Accumulator resetting process is constant as well, for longer stimuli durations it is more likely to observe the negative TOE than for shorter durations, which is in agreement with the results reported in the literature [14, 27, 32].

1.4 Type A, Type B and the TOE – reconciliation

Sect. 1.3 and earlier works [41, 40, 39] demonstrated that the CCTN is able to simulate the TOE in the form originally described by Allan [2]. Now we will consider the Type A and the Type B phenomena and show that under certain conditions they emerge in the CCTN.

Fig. 3 shows the response functions resulting from a sample 2-AFC simulation experiment. As it can be seen, the two thick lines for (C,S) and (S,C) orders intersect above the PSE point, i.e., above P("C>S")=0.5, which means that a small Type A effect is observable. The slope of the psychometric function that represents the (S,C) condition is steeper than that of the function representing the (C,S) condition, which means that the Type B effect is negative. The TOE is the blue line that corresponds to the difference between the P("C>S") values for both presentation orders. In other words, the absolute value of the TOE is the vertical distance between the (S,C) and the (C,S) line. Hence, in this particular example, as the comparison stimulus gets longer, the TOE grows from zero to positive values, then decreases, reaches zero, decreases towards negative values, and finally grows and reaches zero again. As the figure demonstrates, when the set of pairs of experimental stimuli is arranged such that the duration of one of the stimuli is constant (standard stimulus), the ISI is constant and the only variable factor is the duration of the comparison stimulus, then:

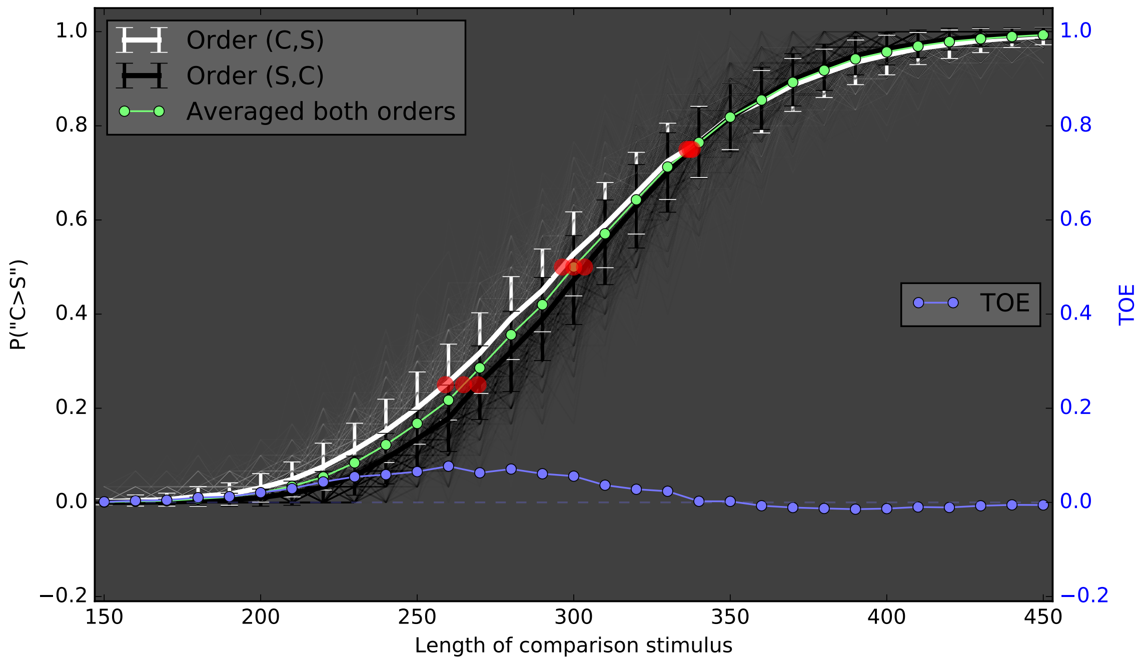

Figure 3: Exemplary psychometric functions and the TOE line produced by the CCTN in a sample 2-AFC simulation experiment with an arbitrarily configured model. The standard stimulus lasted 300 simulation steps and the comparison stimuli varied from 150 to 450 steps. Red dots indicate the points calculated for P("C>S") values of 0.25, 0.5 and 0.75. Thin semi-transparent white and black lines are individual simulation runs (960 in total). The two thicker lines, white and black, represent averaged values for each presentation order, and vertical bars are their standard deviations. The green line represents all values averaged (independently of the order of presentation).

- the TOE depends on the duration of the comparison stimulus,
- the difference between the slopes of the (S,C) and (C,S) functions reflects the changes in the TOE,
- the TOE value can be calculated as the vertical distance between the lines, and the polarity of the TOE is determined by the relative position of these functions: if the (C,S) value is higher then the TOE is positive; otherwise it is negative,
- one of the reasons for the difference in slopes of the response functions for both orders is the fact that as the duration of the comparison stimulus increases, more remnant information in the Accumulator adds up to the standard stimulus pulses in the (C,S) order, which potentially decreases P("C>S"). This is not the case in the (S,C) order,
- additionally, the bias value in the Accumulator also influences the slopes of both response functions,
- hence, according to the CCTN model, the indicators of order effects (the slopes and distances between PSE points of the response functions) result from the interplay between the stimuli magnitude, the bias value in the Accumulator, and the Accumulator Reset Rate. The CCTN predicts that the ISI also influences these indicators of order-effects.

A simulation study was performed to provide a more detailed view of the CCTN predictions about the TOE, the Type A and the Type B effects and their indicators such as the slopes of the response functions and the PSE values. The goal of this study was to demonstrate that the CCTN predicted the three abovementioned order effects as originating from one general timing mechanism. In this study, various stimuli ranges, bias values in the Accumulator and the Accumulator Reset Rates were tested.

Note that the usual procedure in psychophysical studies is to fit some psychometric (often logistic) function to the collected data. This kind of processing of data may isolate the original stimuli used in an experiment from the general view of the changes of P("C>S") across a wide range of durations of comparison stimuli. Moreover, focusing on a usually small range of experimental stimuli may obscure the general explanation of the mechanism responsible for the TOE and the Type A and Type B effects. Since in this work we are experimenting with a simulation model, we can perform a large number of experiments and obtain stable averaged results as well as their variance, and these characterize the behavior of the model with (arbitrarily) high precision. Therefore we decided to work on such raw data and did not employ fitted psychometric functions to avoid any loss of information or patterns potentially existing in the data. This means that there is less post-processing of experimental data in this work than usually seen in the literature where human subjects are involved in experiments.

1.5 Weber's Law

One of the important theoretical constructs in psychophysics is the just-noticeable difference (JND) [24, 44]. It is the amount of stimulation that should be added to some initial/standard stimulus in order for a human to recognize a difference in this stimulus. The JND is a measure of sensitivity of subjects to changes of the stimulation level. As stated in Sect. 1.2, a measure often used to represent the JND is the difference limen. It has been noted by Weber and Fechner that the ratio of the JND to the standard stimulus (called the Weber fraction) should be constant in the perceiving subjects – this property has been called Weber's Law. Subsequent research in psychophysics revealed that Weber's Law does not hold in general [21, 23, 60, 62, 24], yet in selected ranges of stimuli human participant produce results that conform to this law. In timing research, due to various, often methodological reasons, the law is sometimes replaced by a property which is often called the scalar property – the coefficient of variation of judgments made by subjects during timing tasks is constant [21, 16, 36]. While this property was also found to not hold in its strict form in the full range of stimuli durations [45, 75, 36, 3, 38], it is still often examined to provide information on perceptual sensitivity of subjects.

The CCTN has a hardwired mechanism – the Scalar Variance Module – that induces scalar timing. It has been shown [41] that this module is not able to make the CCTN yield a perfect scalar timing – in the range of very short stimuli the coefficient of variation is higher than for longer durations, which, as mentioned in Sect. 1.5, is in agreement with the experimental results reported in the literature. However, so far the behavior of the CCTN was not tested in terms of Weber fractions expressed as the $\frac{\text{JND}}{\text{S}}$ ratio (S denotes the duration of the standard stimulus). The predictions of the CCTN regarding Type A, Type B, the TOE and Weber's Law phenomena will be discussed in the following sections.

2 Methods

For simulation experiments, the Framsticks software has been employed [43, 42, 34, 26, 48, 58]. This simulator is available for most desktop and mobile platforms, and is capable of

<table>
<tr><th>Experiment #</th><th>CCTN variant</th><th>Standard stimulus, S</th><th>ISI</th></tr>
<tr><td>Experiment 1</td><td>high ARR, high ABV
high ARR, low ABV
low ARR, high ABV
low ARR, low ABV</td><td>300–700 ss</td><td rowspan="2">1000 ss</td></tr>
<tr><td>Experiment 2</td><td rowspan="2">high ARR, low ABV</td><td>300–1600 ss</td></tr>
<tr><td>Experiment 3</td><td>500 ss</td><td>700–5000 ss</td></tr>
</table>

Table 1: Parameter values in the three simulation experiments. Both standard stimuli and ISI are sampled every 100 ss (simulation steps) within the specified ranges.

simulating artificial neural networks of arbitrary topology. The way individual neuron types work can be defined by writing script files in the FramScript programming language. This language also allows to control the overall behavior of the simulation environment and the logic of the experiments. Framsticks is also available as a library that can be used in other programming languages such as Python.

Let us call a CCTN model characterized by selected parameter values a *variant*. Variants of the CCTN were determined by two parameters: the Accumulator resetting rate, ARR (the value taken away from the stored Accumulator signal in each simulation step during the clearing process) and the Accumulator Bias Value, ABV (the default value of the signal stored in the Accumulator). Each variant was characterized by the ARR value (high: 0.0024 or low: 0.0020) and the ABV value (high: 15 or low: 10). These values were chosen so that the CCTN model produced response functions that differ for each variant, yet still resembled human response functions. A more detailed description of CCTN modules and the analysis of their parameters is provided in [41].

All the simulation experiments reported in this work shared the same structure:

- Each CCTN variant was simulated 960 times.
- Denoting the duration of the standard stimulus as S, the range of durations of comparison stimuli C was $[\mathrm{S}-150, \mathrm{S}+150]$ simulation steps sampled every 10 steps, yielding 31 distinct stimuli pairs that share the same S.
- The presentation of each pair of stimuli was preceded by 2750 ss (simulation steps) of the trial onset period, and was followed by 3000 ss of the response period. Both of these periods were devoid of any stimulation and were included to allow the network to return to the initial state between trials.
- Each of these 31 stimuli pairs was presented to CCTN in two orders (C,S) and (S,C), resulting in a collection of 62 ordered stimuli pairs (the case where a comparison stimulus C is equal to the standard S appears in the collection twice).
- In a single simulation, each ordered pair from such a collection was presented to a CCTN variant 30 times in a randomized order.
- Therefore, each of the 960 simulations of a CCTN variant was a sequence of $31 \times 2 \times 30$ ordered pairs of stimuli durations that all shared a single standard stimulus duration, S.

Three kinds of experiments were performed as summarized in Table 1. The goal of all these experiments was to demonstrate the Type A and Type B effects and their relation to the TOE and the Weber's Law in the CCTN. Each of the three experiments focused on a different aspect of the modeled comparison process. Experiment 1 examined the influence of crucial internal timing mechanisms which induce the TOE effect on the other psychophysical phenomena. As the selected CCTN parameters significantly influence the magnitude of the TOE, it was important to verify how the parameter values would influence the Type A and Type B effects as well as the dynamics of the Weber fraction. Experiments 2 and 3 focused on physical aspects of external stimulation: the objective duration of the experimental stimuli (standard and comparison) and the time interval between the compared stimuli (ISI). Experiments 2 and 3 explained how the modeled timing mechanism accounted for the varying characteristic of the external stimulation to yield particular response functions. Together, the three experiments provided an extensive overview of psychophysical timing mechanisms and how they depend on the internal and external factors such as the objective duration of the experimental stimuli and the ISI. They also revealed the relationships between important psychophysical phenomena displayed by the CCTN – the TOE, the Type A and B effects and the Weber's Law.

The main characteristic that was measured for each stimulus pair in each experiment was the frequency of the answer "the comparison stimuli is longer than the standard", i.e., P("C>S"). Based on the P("C>S"), the PSE, the DL, as well as Type A, Type B and the TOE values were calculated. Additionally, to examine the sensitivity of the CCTN for each combination of parameters, the Weber fraction (WEB) was calculated as the $\mathrm{WEB} = \frac{\mathrm{JND}}{\mathrm{S}}$ ratio, where JND is represented by the difference limen (DL) and S stands for the duration of the standard stimulus.

3 Results and Discussion

As there are multiple parameters and relationships collected and analyzed in the experiments, for the sake of clarity we present separately the results with their basic analyses (Sect. 3.1) and comprehensive discussions (Sect. 3.2). Moreover, the Type A effect, the Type B effect, Weber fractions and the TOE are treated in separate subsections.

3.1 Results

The simulation experiments yielded interesting observations concerning response functions of different CCTN variants. Fig. 4 presents sample response functions for the high ARR and the low ABV for three standard stimuli durations in Experiment 1. As it can be seen, in this particular setting the relative position of (S,C) and (C,S) lines changes for different standard stimuli durations, which results in the Type A effect – a positive Type A effect for short standard stimulus durations, almost zero for medium durations, and negative for the long durations. As the relative placement of those lines also determines the TOE, it is visible that when the standard stimulus gets longer, the TOE becomes overall more and more negative as indicated by the blue lines in the figure. The points of intersection of the (C,S) and (S,C) response functions correspond to zero TOE. The Type B effect also changes its magnitude because of different slopes of the white and black lines in different plots. However, as changes in slopes are accompanied by shifts in relative positions of the (C,S) and (S,C) response functions, in order to draw more comprehensive conclusions a more careful examination of the difference limens will be provided in further sections.

Green lines in the plots represent the aggregated, averaged results for each combination of experimental parameter values. Interestingly, the points of subjective equality for these lines seem a little confusing. In each of the plots in Fig. 4, the PSE for the green line is close to the standard stimulus duration; however, apart from the case when the standard stimulus lasts 500 ss, the PSE values for the (S,C) and (C,S) orders are different. This reveals a bias in

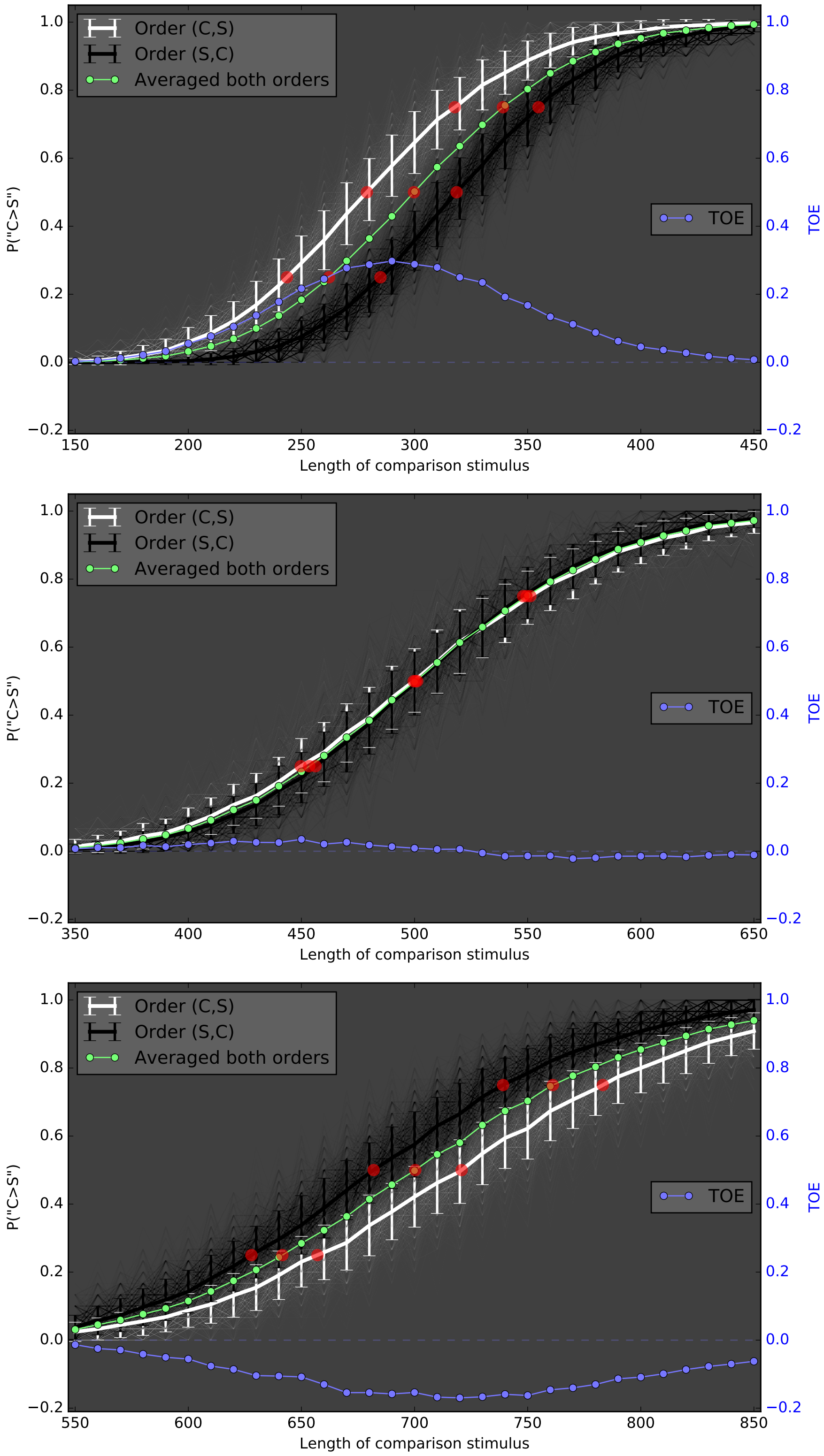

Figure 4: Sample psychometric functions and the TOE lines obtained in Experiment 1 with the ARR high and the ABV low. The top, middle and bottom plots present the results for standard stimuli lasting 300, 500 and 700 ss (simulation steps), respectively. The red dots depict the points calculated for P("C>S") values equal to 0.25, 0.5 and 0.75.

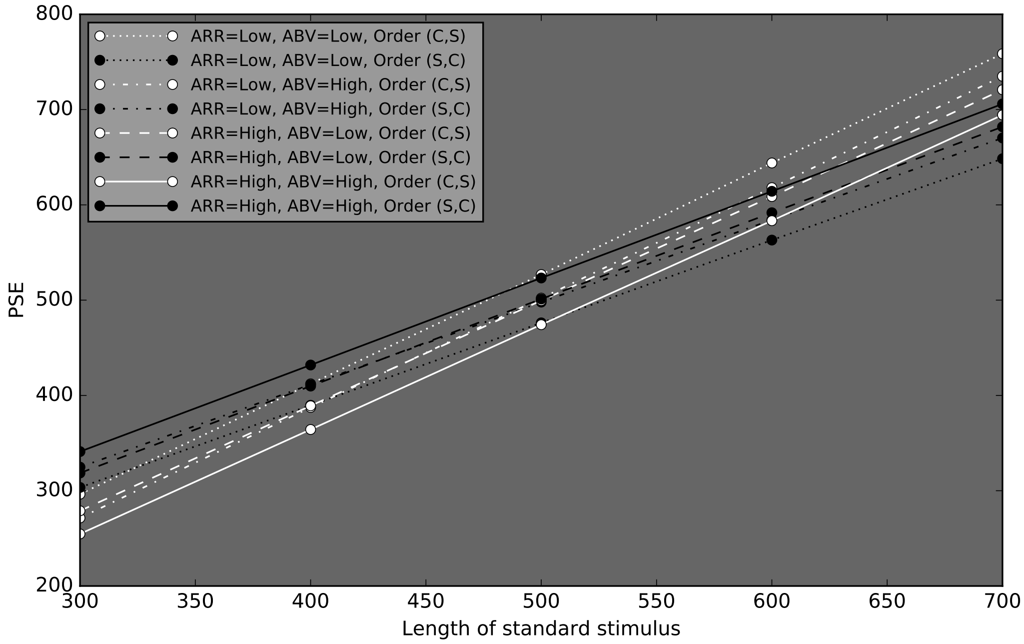

Figure 5: The PSE values for a range of standard stimulus durations in Experiment 1. Distinct line styles (dotted, dash-dotted, dashed, or solid) represent PSE values for different values of the Accumulator Reset Rate (ARR) and the Accumulator Bias Value (ABV) parameters. The color of lines (white or black) denotes the order of presentation.

the response functions that is responsible for shifting the PSE values horizontally for the two orders of presentation. The bias is actually induced by the TOE mechanism. Averaging the responses over the orders of presentation may cover up this bias, yielding a false subjective equality indicator in the 2-AFC paradigm [73].

The following sections present more thorough analyses of the Type A and B phenomena, Weber fraction and the TOE.

3.1.1 The point of subjective equality (PSE)

Experiment 1 yielded linear PSE trends for each combination of the ARR and ABV values, as Fig. 5 shows. What is not visible in this plot is that the PSE values for the averaged response functions (these have been averaged independently of the order of presentation) are nearly identical to the duration of standard stimuli for all the combinations of parameter values in Experiment 1. There is a clear difference between the slopes of the lines in both orders of presentation: the (S,C) order yields less steep lines, and the (C,S) order yields steeper lines. The difference between slopes changes the difference between PSE values for both orders and their corresponding ARR and ABV values. For the shortest standard stimuli, the largest difference is between the PSE values for (S,C) and (C,S) orders for the high ARR and the high ABV (two solid lines), and the lowest difference is for the low ARR and the low ABV (two dotted lines). This relation is inverted for the longest standard stimuli.

The PSE values obtained in Experiment 2 are shown in Fig. 6. The plot reveals linear trends of increase of the PSE values as the magnitude of the standard stimulus grows. The increase of the PSE values is the highest for the (C,S) order of presentation and the lowest for the (S,C) order. The PSE for the averaged response functions yields a function that is nearly equal to (and visually indistinguishable from) the average of PSE values for both

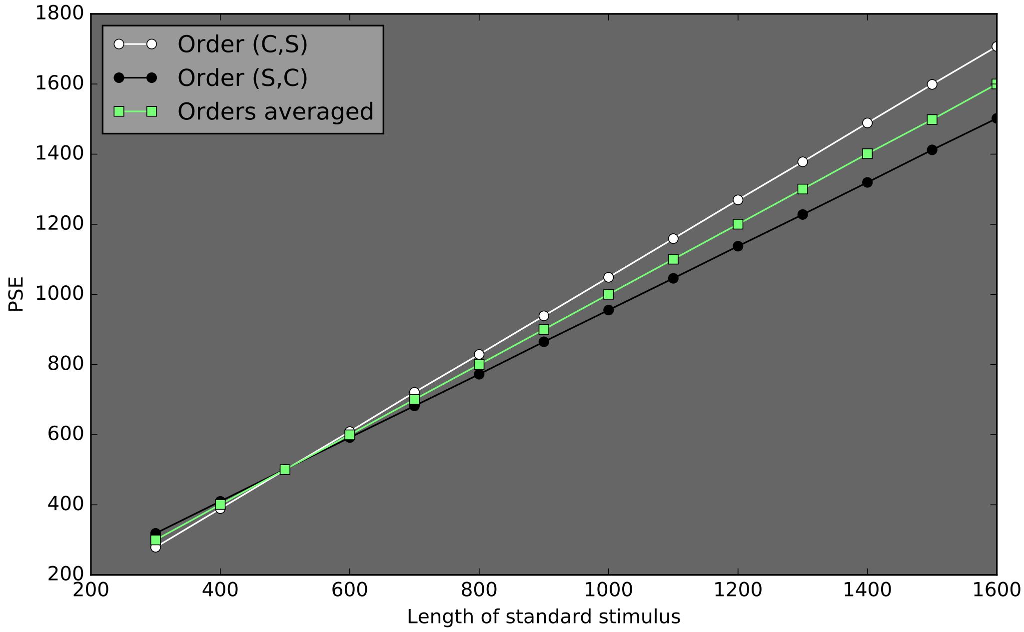

Figure 6: The PSE values in Experiment 2 for a wide range of standard stimulus durations. Accumulator Reset Rate (ARR) was high and Accumulator Bias Value (ABV) was low.

orders. The averaged PSE is also nearly equal to the duration of the standard stimulus, and the intersection of the three PSE functions is close to the point of objective equality $(500, 500)$.

Fig. 7 demonstrates the PSE values in Experiment 3 for a range of ISI magnitudes and the standard stimulus lasting 500 ss. The (S,C) and (C,S) PSE values exhibit non-linear dependencies of the ISI and are nearly perfect mirror reflections of each other, and the PSE values calculated from averaged response functions from both orders are independent from the ISI value. All lines intersect at the point where the ISI value is approximately 1000 ss, so there is no Type A effect for this particular ISI value in this experiment. When ISI grows starting from low values, the values of PSE increase or decrease (depending on the order of stimuli presentation), then the trends reverse, and ultimately, for high ISI values, the PSE values approach zero and stabilize at zero – resulting again in the lack of the Type A effect.

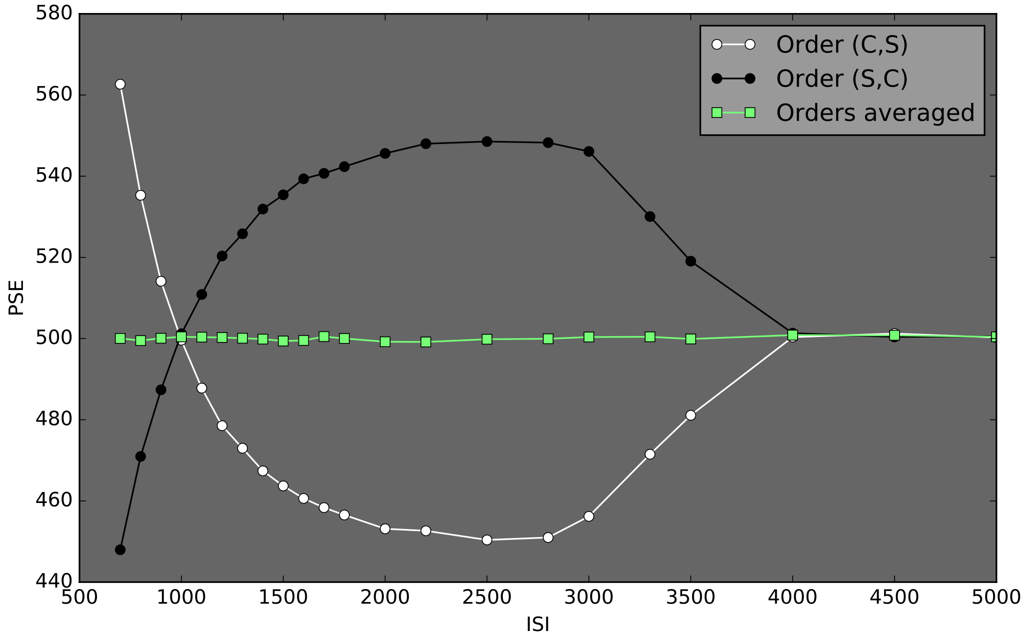

Figure 7: The PSE values in Experiment 3 for a range of inter-stimulus intervals (ISI) for the standard stimulus lasting 500 ss. Accumulator Reset Rate (ARR) was high and Accumulator Bias Value (ABV) was low.

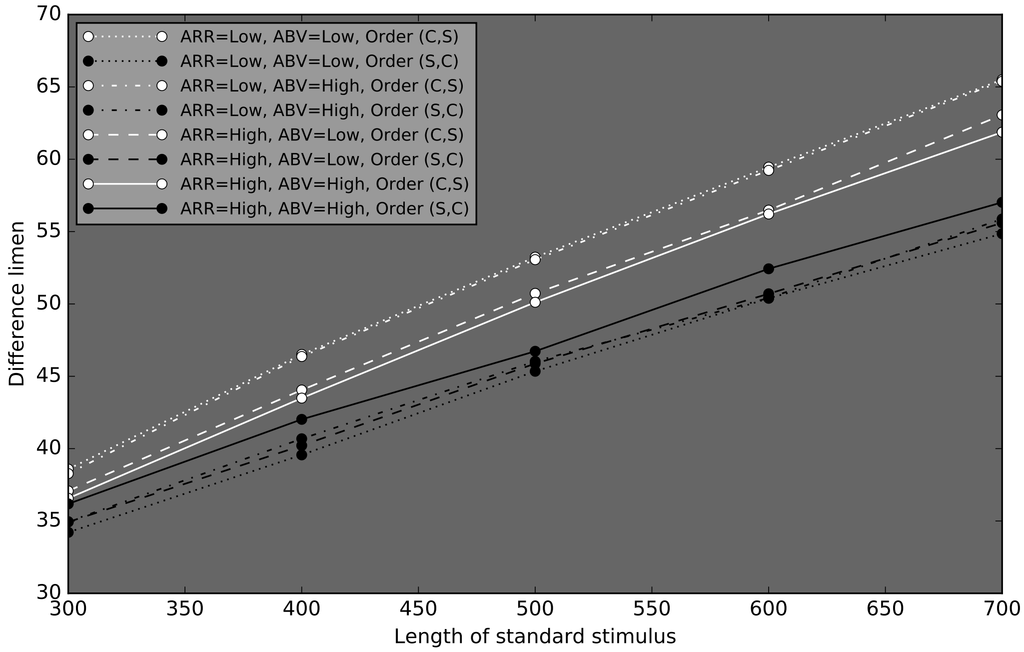

Figure 8: The difference limen (DL) for a range of standard stimulus durations in Experiment 1. Distinct line styles (dotted, dash-dotted, dashed, or solid) represent DL values for different values of the Accumulator Reset Rate (ARR) and the Accumulator Bias Value (ABV) parameters. The color of lines (white or black) denotes the order of presentation.

3.1.2 Difference limen (DL)

The values of the difference limen obtained in Experiment 1 are presented in Fig. 8. Difference limens for the (S,C) order are lower and, as the duration of the standard stimulus grows, increase more slowly than difference limens for the (C,S) order.

Fig. 9 presents nearly linear DL trends resulting from Experiment 2, with DL values increasing with the duration of the standard stimulus. Since the average DL values are calculated from the averaged individual response functions and depend on their slope, and the slope of the averaged functions may differ greatly from the slopes of the order-dependent response functions, the averaged DL line does not lie between the DL values calculated for the two orders. Longer standard stimuli make the difference between DL values for different presentation orders larger.

The difference limens obtained in Experiment 3 are shown in Fig. 10. For ISIs in the range of 700–1500 ss, the DL as a function of ISI for the (S,C) order is decreasing and for the (C,S) order it is increasing. For longer ISIs, both DLs stabilize and later converge. DL values for the averaged series do not depend in any obvious way on the DL values for both presentation orders – as noted above, the averaged DL values depend on the slopes of averaged response functions that may be very different from the slopes of both order-dependent response functions.

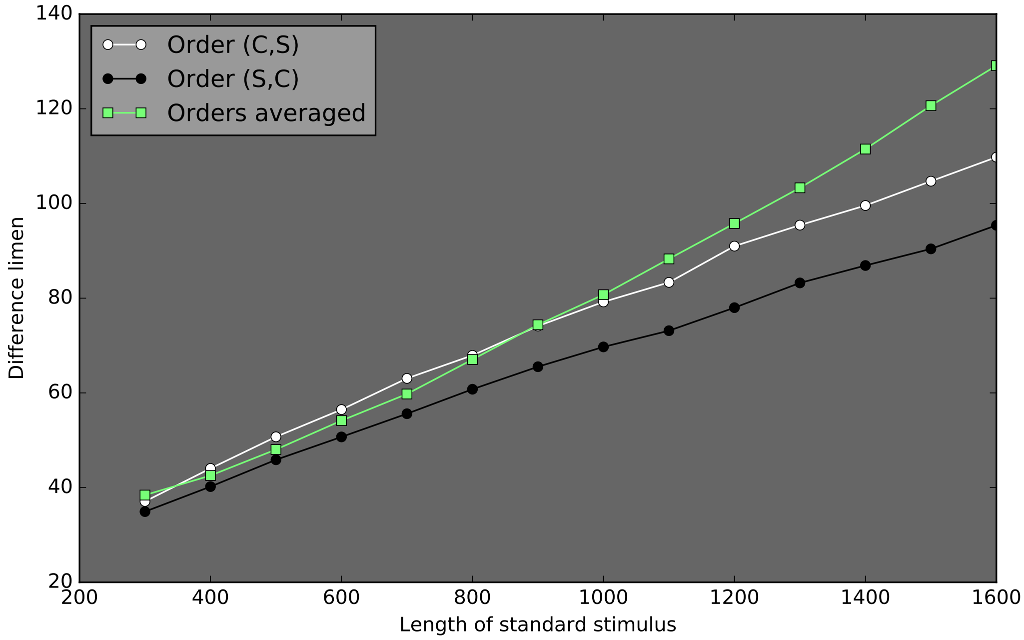

Figure 9: The DL values in Experiment 2 for a wide range of standard stimulus durations. Accumulator Reset Rate (ARR) was high and Accumulator Bias Value (ABV) was low.

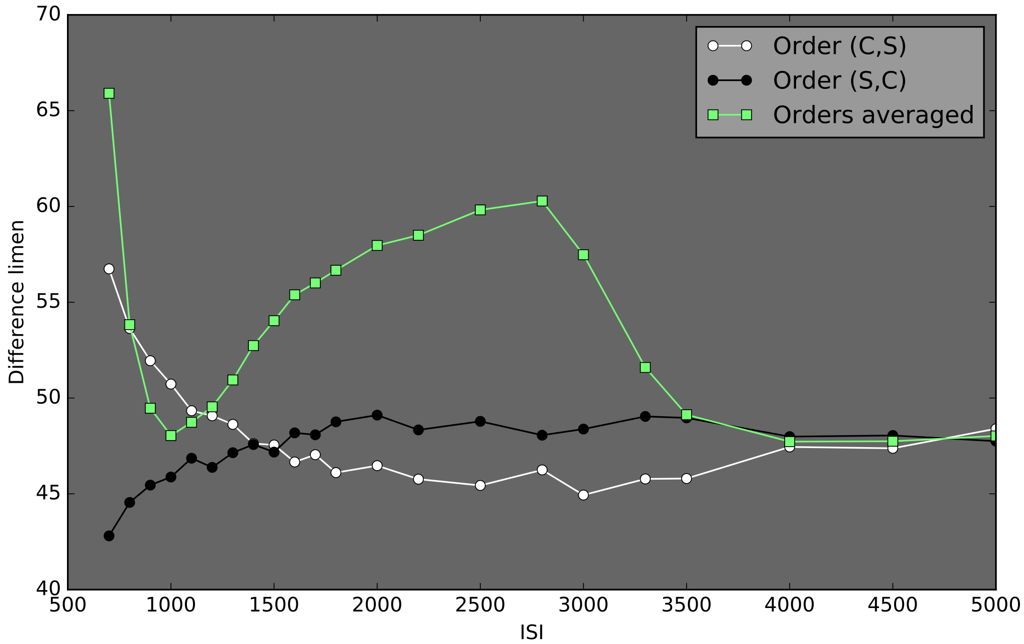

Figure 10: The DL values in Experiment 3 for a range of inter-stimulus intervals (ISI) for the standard stimulus lasting 500 ss. Accumulator Reset Rate (ARR) was high and Accumulator Bias Value (ABV) was low.

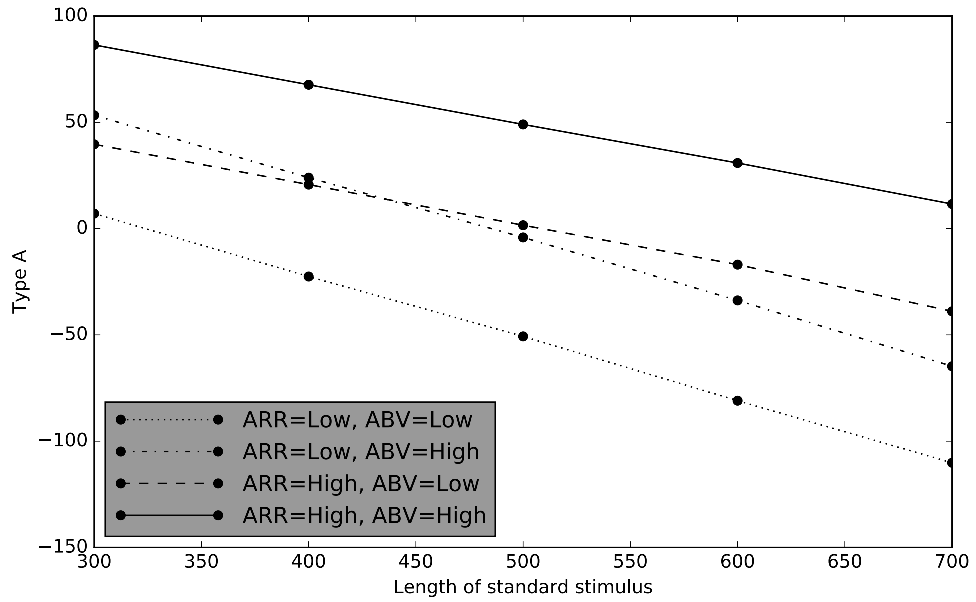

Figure 11: Type A values for a range of standard stimulus durations in Experiment 1. Distinct line styles (dotted, dash-dotted, dashed, or solid) represent the values of the Type A effect for different values of the Accumulator Reset Rate (ARR) and the Accumulator Bias Value (ABV) parameters.

3.1.3 Type A and Type B

The Type A values obtained in Experiment 1 (shown in Fig. 11) drop steadily as a function of the standard stimulus durations for each combination of the ARR and ABV parameter values. The four trends shown in the plot are nearly perfectly linear (their correlation coefficients r are -0.99996, -0.99986, -0.99954, and -0.99997, respectively). Moreover, it is possible to predict extremely accurately the amount of the Type A effect based on the values of ARR and ABV. It can be seen that the slopes of the four Type A functions depend almost entirely on the ARR. The slope coefficients can be approximated as a linear combination of both ARR and ABV with a very high accuracy (the correlation coefficient is 0.9990): $\mathrm{TypeA}_{\mathrm{slope}} = 256.831 \cdot \mathrm{ARR} + 0.0007 \cdot \mathrm{ABV} - 0.8161$. Dropping the dependency on ABV still results in a very good approximation, $\mathrm{TypeA}_{\mathrm{slope}} = 256.831 \cdot \mathrm{ARR} - 0.807$ (the correlation coefficient drops to 0.9984). Similarly, the intercept coefficient of Type A as a linear combination of ARR and ABV depends almost entirely on ABV, and can be approximated as $\mathrm{TypeA}_{\mathrm{intercept}} = 9.0642 \cdot \mathrm{ABV} + 6.1562$ (the correlation coefficient is 0.9984). Therefore in Experiment 1, the amount of the Type A effect as a function of standard stimulus duration ("Standard"), ARR and ABV can be highly accurately predicted as $\mathrm{TypeA} = (256.831 \cdot \mathrm{ARR} - 0.807) \cdot \mathrm{Standard} + 9.0642 \cdot \mathrm{ABV} + 6.1562$.

For each duration of the standard stimulus, the highest values of Type A are obtained by the networks with high ARR and high ABV, while the lowest values are obtained for the networks with low ARR and low ABV. As for the low-high and high-low CCTN variants, for the shorter durations of the standard stimuli, the PSE values are larger for low ARR and high ABV, while for longer durations this relation is reversed. Even though the Type A values are the direct consequences of the PSE values presented in Sect. 3.1.1, the Type A values precisely show how different combinations of CCTN parameter values (that constitute different variants of the network) influence relations between order-dependent PSE as

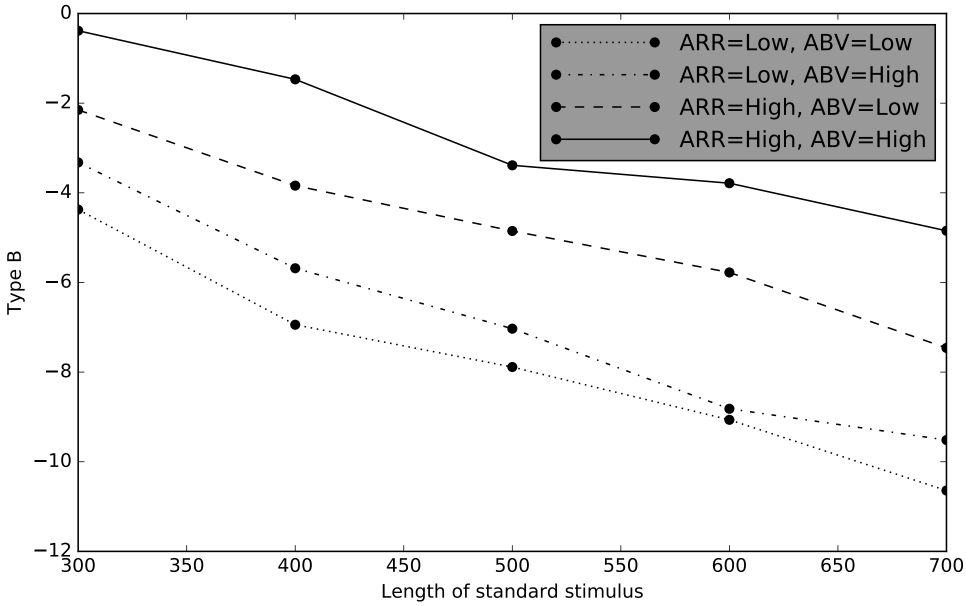

Figure 12: Type B values for a range of standard stimulus durations in Experiment 1. Distinct line styles (dotted, dash-dotted, dashed, or solid) represent the values of the Type B effect for different values of the Accumulator Reset Rate (ARR) and the Accumulator Bias Value (ABV) parameters.

functions of the duration of standard stimulus.

Fig. 12 shows the Type B values obtained from Experiment 1. Type B results are less stable than the results for Type A shown in Fig. 11, note however a much narrower range of the values of the Type B effect. Just as Type A, for each duration of standard stimulus, the Type B effect is the strongest for high values of ARR and ABV, and the weakest for low values of these two parameters. The four trends shown in the plot are highly linear (their correlation coefficients r are -0.985, -0.986, -0.993, and -0.982, respectively). Similarly to Type A above, it is possible to predict the amount of the Type B effect based on the values of ARR and ABV. The slope coefficients of the four Type B functions can be approximated as $\text{TypeB}_{\text{slope}} = 7.9705 \cdot \text{ARR} - 0.0316$ (the correlation coefficient is 0.9466). The intercept coefficient of Type B as a linear combination of ARR and ABV can be approximated as $\text{TypeB}_{\text{intercept}} = 4847.471 \cdot \text{ARR} + 0.2723 \cdot \text{ABV} - 12.8824$ (the correlation coefficient is higher than 0.9999, and both ARR and ABV are relevant). Analogously to Type A, in Experiment 1 the amount of Type B effect as a function of standard stimulus duration ("Standard"), ARR and ABV can be described as $\text{TypeB} = \text{TypeB}_{\text{slope}} \cdot \text{Standard} + \text{TypeB}_{\text{intercept}}$.

Fig. 13 presents the values of the Type A effect in Experiment 2. Type A decreases linearly as the duration of the standard stimulus increases; no Type A effect is observed for the standard stimulus close to 500 simulation steps. A linearly decreasing trend of Type A is not surprising given linear trends of PSE values for both orders of presentation. Fig. 14 shows linearly decreasing Type B values in Experiment 2. Again, they are less stable than Type A, but also the variability of the Type B values is much lower, as demonstrated by the much narrower range.

The Type A and Type B values obtained in Experiment 3 (shown in Figs. 15 and 16) initially grow non-linearly with the increasing ISI, and then drop to zero. As previously, the range of the Type B values is narrower and its random fluctuations are more visible.

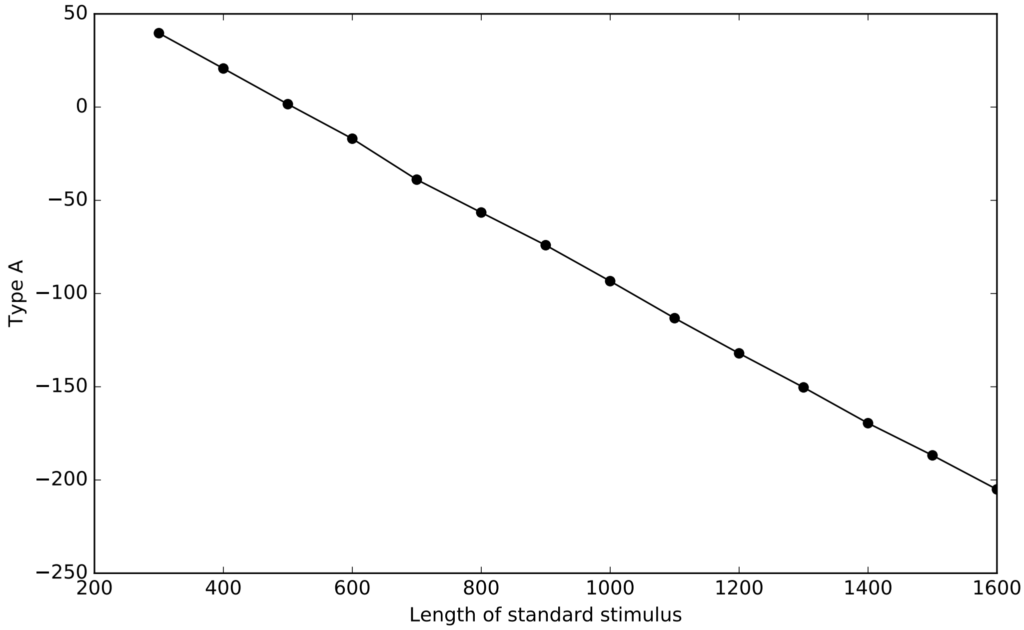

Figure 13: The values of the Type A effect in Experiment 2 for a wide range of standard stimulus durations. Accumulator Reset Rate (ARR) was high and Accumulator Bias Value (ABV) was low.

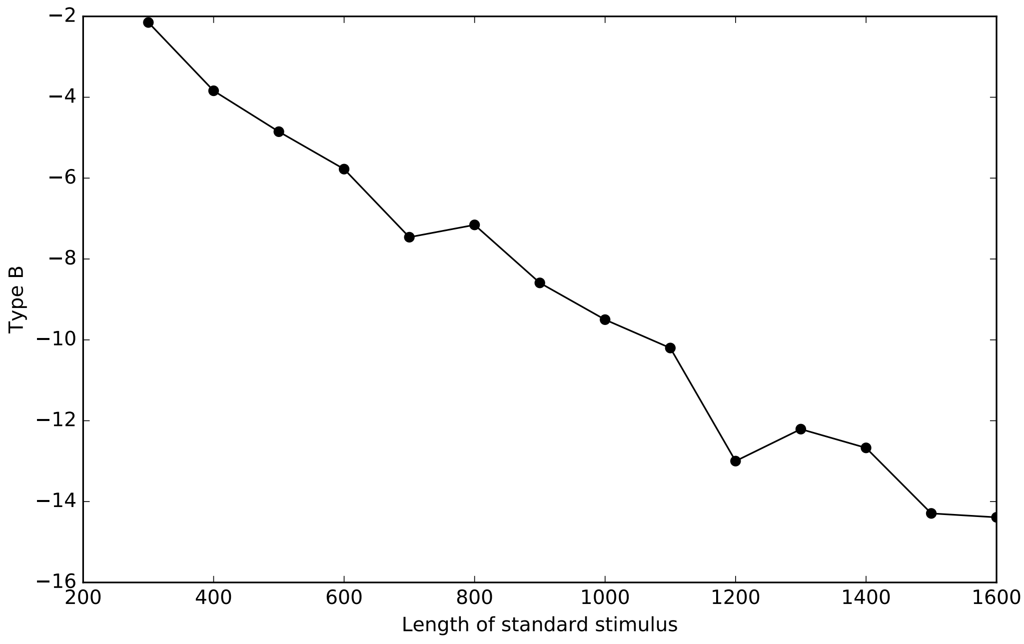

Figure 14: The values of the Type B effect in Experiment 2 for a wide range of standard stimulus durations. Accumulator Reset Rate (ARR) was high and Accumulator Bias Value (ABV) was low.

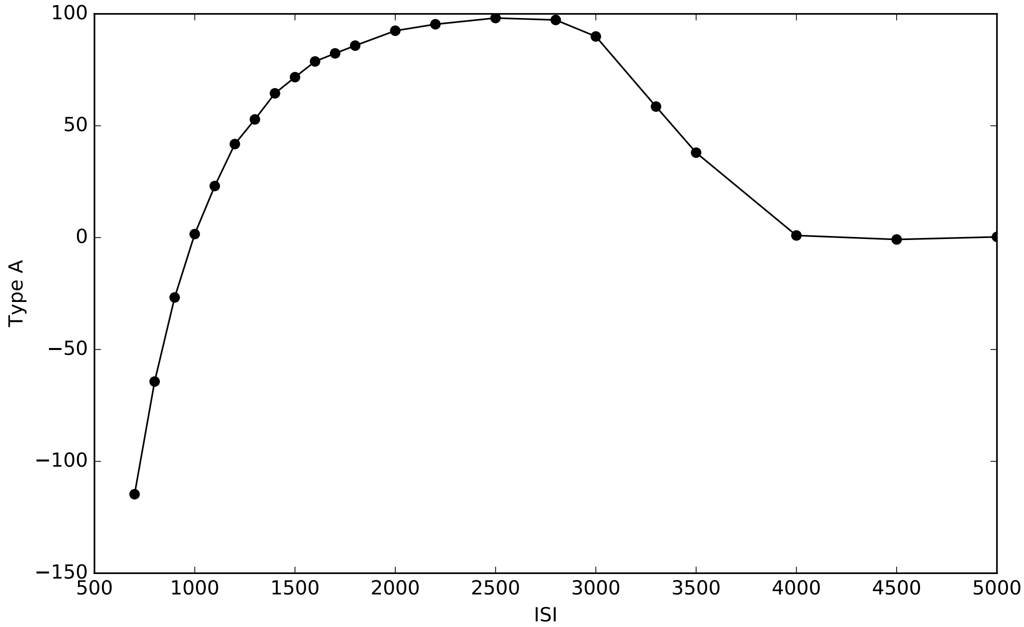

Figure 15: The values of the Type A effect in Experiment 3 for a range of inter-stimulus intervals (ISI) for the standard stimulus lasting 500 ss. Accumulator Reset Rate (ARR) was high and Accumulator Bias Value (ABV) was low.

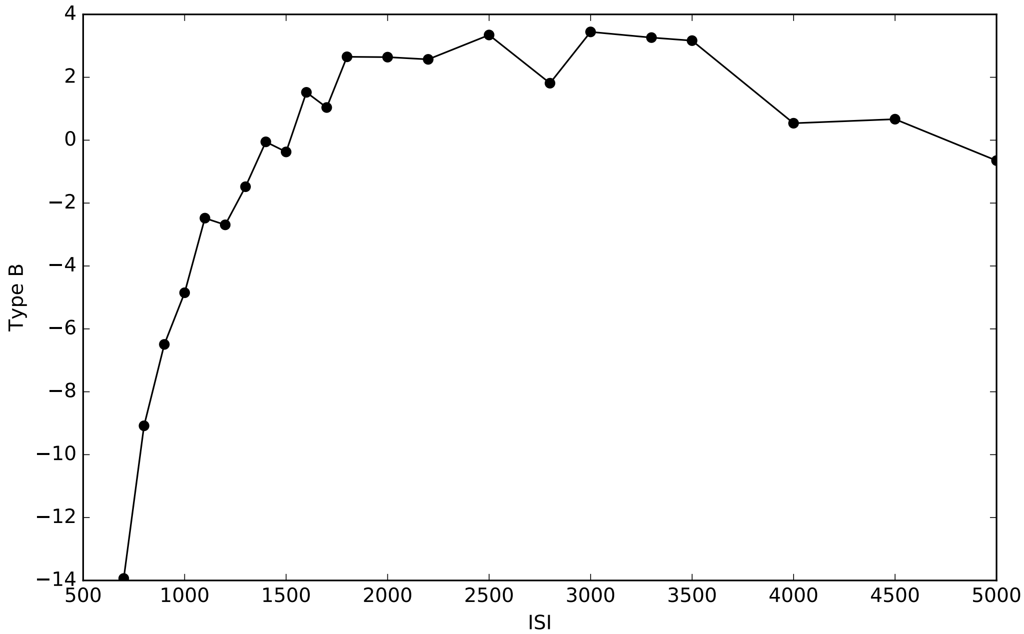

Figure 16: The values of the Type B effect in Experiment 3 for a range of inter-stimulus intervals (ISI) for the standard stimulus lasting 500 ss. Accumulator Reset Rate (ARR) was high and Accumulator Bias Value (ABV) was low.

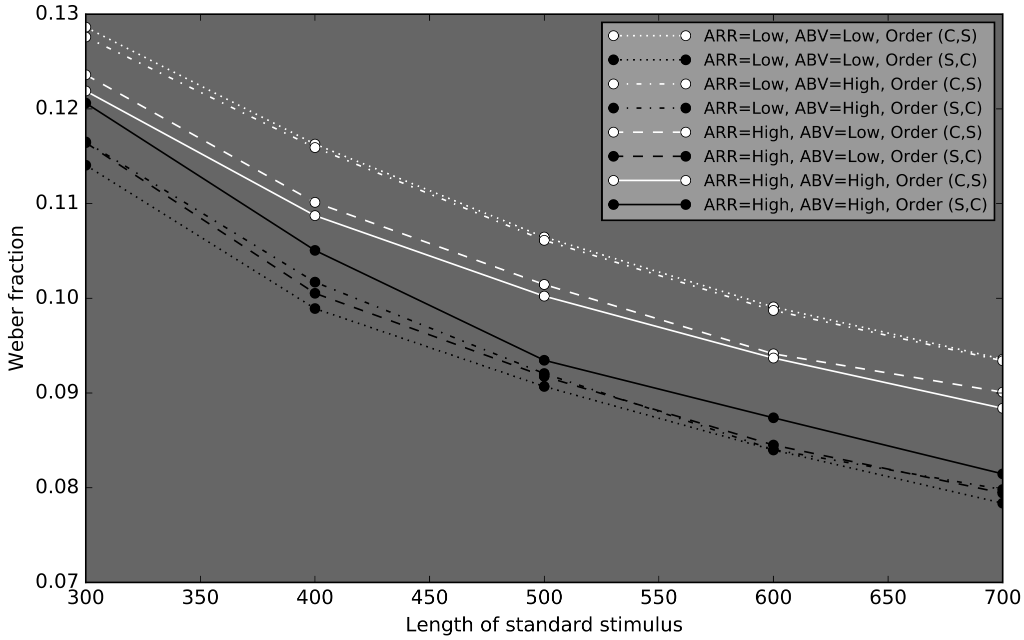

Figure 17: The Weber fraction (WEB) for a range of standard stimulus durations in Experiment 1. Distinct line styles (dotted, dash-dotted, dashed, or solid) represent WEB values for different values of the Accumulator Reset Rate (ARR) and the Accumulator Bias Value (ABV) parameters. The color of lines (white or black) denotes the order of presentation.

3.1.4 Weber fraction

The Weber fraction values obtained in Experiment 1 are shown in Fig. 17. In each network variant and presentation order, the WEB values decrease as the standard stimulus becomes longer. Lower WEB values are obtained for the (S,C) order than for the (C,S) order, which is due to the differences between DLs described in Sect. 3.1.2.

The values of the Weber fraction calculated from the data obtained in Experiment 2 are shown in Fig. 18. The plot shows that Weber fraction decreases as the standard stimulus increases, and for the averaged orders of presentation, it stabilizes sooner than for the (C,S) and (S,C) orders. This once again indicates that for the Weber fraction, the averaged case is not a simple aggregation of the (C,S) and (S,C) values.

Fig. 19 showns the WEB values in Experiment 3. As the duration of the standard stimulus is constant for all calculated WEB values, the trends visible here are identical to the DL trends in Fig. 10.

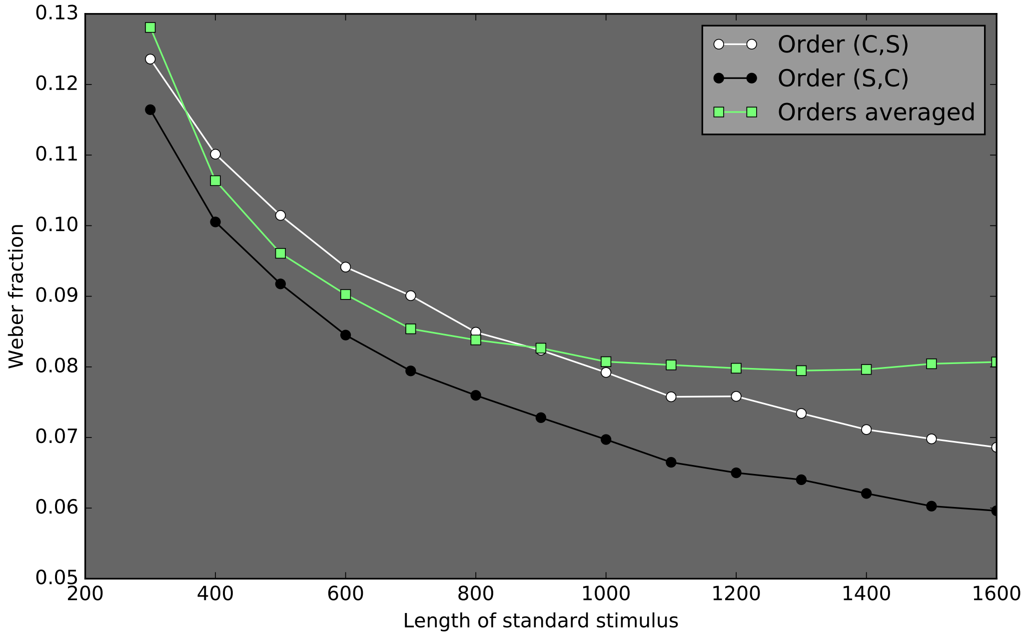

Figure 18: The values of the Weber fraction in Experiment 2 for a wide range of standard stimulus durations. Accumulator Reset Rate (ARR) was high and Accumulator Bias Value (ABV) was low.

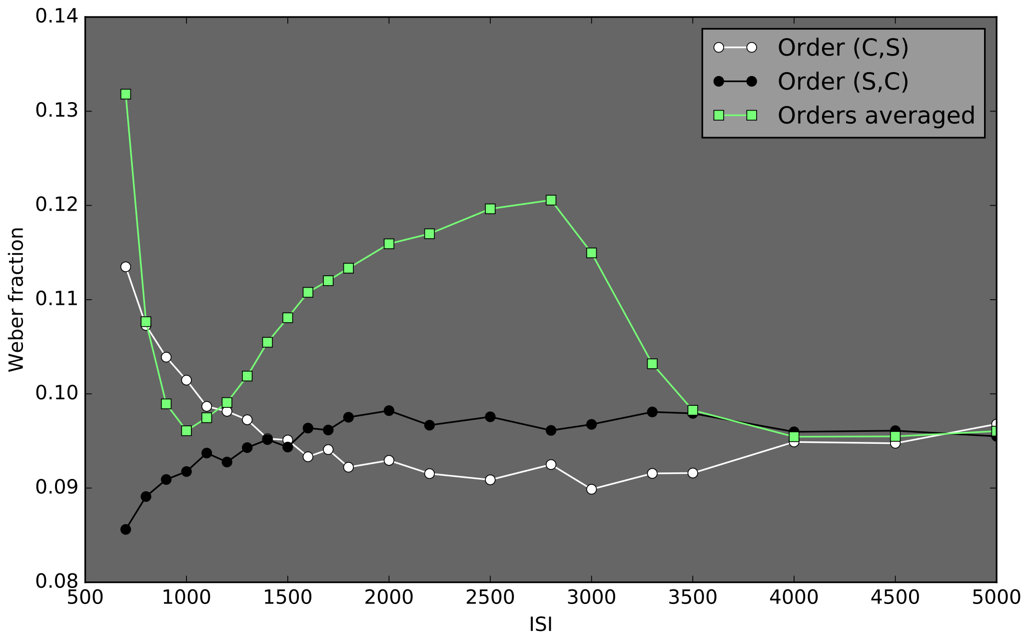

Figure 19: The values of the Weber fraction in Experiment 3 for a range of inter-stimulus intervals (ISI) for the standard stimulus lasting 500 ss. Accumulator Reset Rate (ARR) was high and Accumulator Bias Value (ABV) was low.

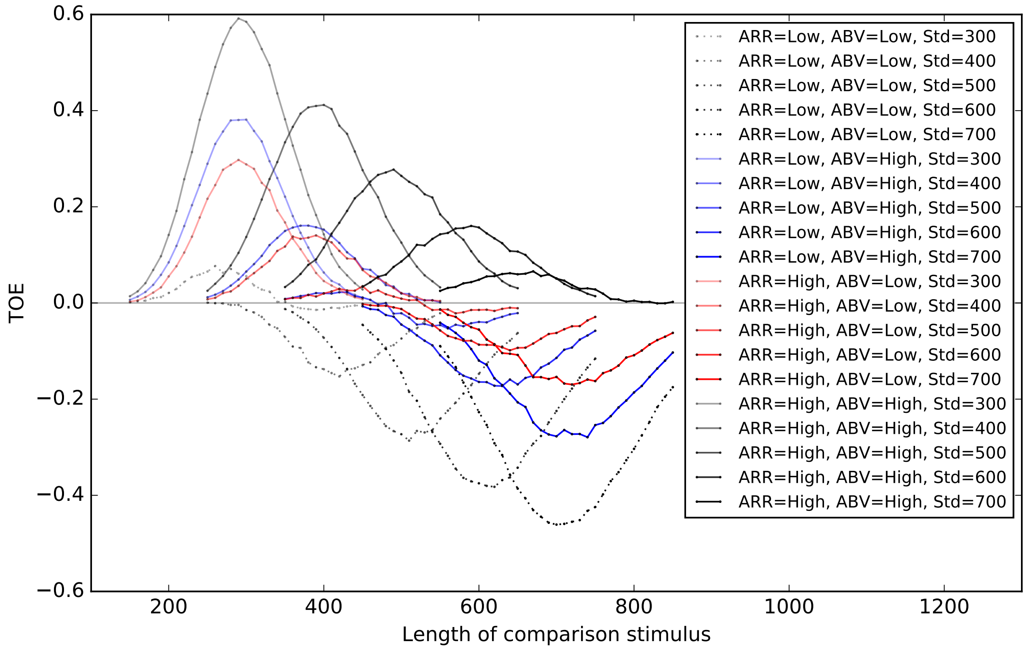

Figure 20: TOE values for various combinations of durations of standard and comparison stimuli in Experiment 1. Distinct line styles (dotted black, solid blue, solid red, solid black) represent TOE for different values of the Accumulator Reset Rate (ARR) and the Accumulator Bias Value (ABV) parameters. Brightness of colors (light to dark) corresponds to the increasing duration of the standard stimulus.

3.1.5 The TOE

The TOE values obtained in Experiment 1 are presented in Fig. 20. The TOE for each stimulus pair is always the highest when the ARR and the ABV values are high and the lowest when the ARR and the ABV are low. Comparing mixed values of these parameters (low-high and high-low), the low ARR and high ABV setting yields more extreme values of TOE. For the shorter durations of the standard stimulus, the TOE is more positive for low ARR and high ABV than for high ARR and low ABV, while for the longer standard stimuli this relation is reversed. There are also two visible properties of peak TOE values. First, for every combination of ARR, ABV and standard stimulus duration, the most extreme TOE occurs when the duration of the comparison stimulus is similar to the duration of the standard stimulus. Second, the peak TOE value reflects the variability and the scaling of the TOE – a low peak value indicates that the TOE has low variability and its values for any duration of the comparison stimulus are close to zero.

Fig. 21 presents TOE values obtained in Experiment 2. Generally, TOE decreases as the duration of the standard stimulus increases – strictly speaking, TOE for the comparison stimuli equally different from the standard becomes more negative as the standard stimulus gets longer. Similarly to Experiment 1 with high ARR and low ABV, the TOE that is the closest to zero and has the lowest variability is achieved for the standard stimulus lasting 500 ss. The TOE series, especially for the longer standard stimuli, are not symmetric with respect to their own peaks. To elaborate, let us consider two sets of stimuli pairs $\{(\mathrm{S}, \mathrm{C}_1), (\mathrm{C}_1, \mathrm{S})\}$ and $\{(\mathrm{S}, \mathrm{C}_2), (\mathrm{C}_2, \mathrm{S})\}$, where the standard stimulus S is fixed in each of these pairs and the two comparison stimuli are equally different from the standard, but one (C_1) *in minus* and the other (C_2) *in plus*. Formally, $\mathrm{C}_1 \leq \mathrm{S} \leq \mathrm{C}_2$ and $\mathrm{S} - \mathrm{C}_1 = \mathrm{C}_2 - \mathrm{S}$.

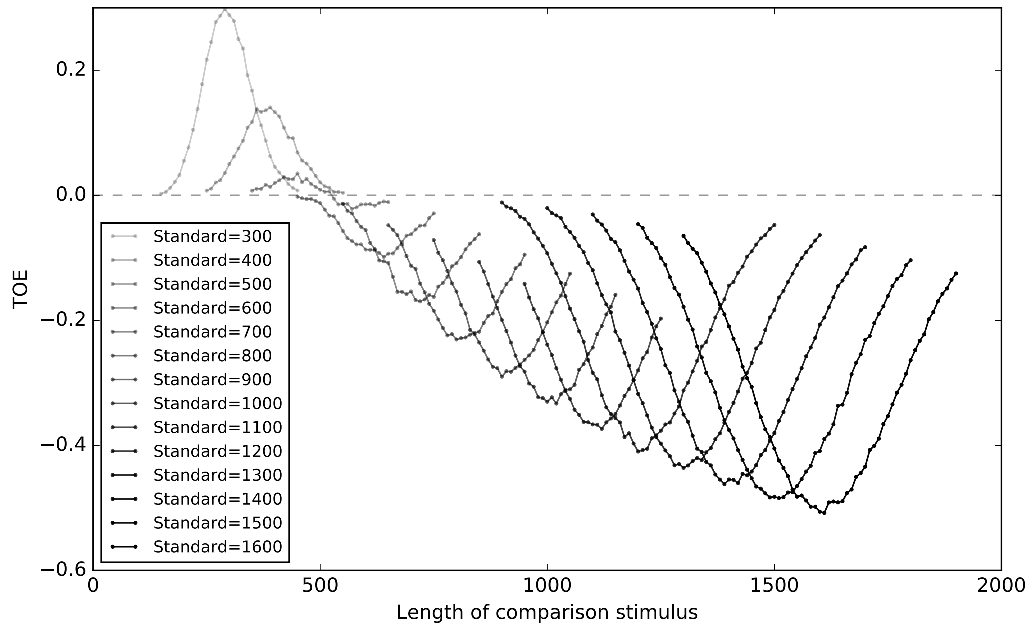

Figure 21: TOE values obtained in Experiment 2. Accumulator Reset Rate (ARR) was high and Accumulator Bias Value (ABV) was low. Each point represents the TOE value for a particular combination of standard and comparison stimuli. Brightness of lines (light to dark) corresponds to the increasing duration of the standard stimulus.

Then, more positive TOE values are obtained for the set of pairs with the shorter comparison stimuli, C_1. This leads to an interesting effect concerning a design of a specific experiment. Let's assume that the experiment is based on a selection of distinct stimuli sets {X,Y}, each set yielding two ordered pairs of stimuli: (X,Y) and (Y,X). If X and Y in each set are related such that $\{X = ai + b, Y = a(i + 1) + b\}$ for some positive constants a and b, where $0 \leq i \leq n - 1$ and n is the number of sets, then the TOE for such a collection would seem to be a steadily decreasing function of the duration of stimulus X, and similarly, of the duration of Y. An example of such an experimental design is the selection of $a = 100$, $b = 600$, and $n = 4$, so the following four sets of stimuli pairs would be employed: $\{X = 600, Y = 700\}$, $\{X = 700, Y = 800\}$, $\{X = 800, Y = 900\}$, and $\{X = 900, Y = 1000\}$. However, another selection of stimuli sets might lead to the observation of different, in fact almost arbitrary trends. This example shows that the results of empirical research might be misleading when a narrow or a somehow biased range of experimental stimuli is considered (cf. [2]). Such results may lead to simplified explanations or conclusions inconsistent with other experimental reports. The power of modeling and simulation tools, as demonstrated here, is that they can provide extensive results and thus reveal relationships which may in turn help protect against oversimplified interpretations of scarce empirical data.

The TOE values in Experiment 3 are shown in Fig. 22. For ISIs shorter than 1000 ss the TOE is negative; for longer ISIs the TOE becomes positive. The shortest ISI durations result in the most negative TOE values. The ISI of approximately 1000 ss yields no TOE – this is where the TOE transitions from negative to positive. The absolute values of the TOE are the highest when the duration of the comparison stimulus is close to the duration of the standard stimulus, i.e., 500 ss. The TOE approaches zero when the difference in durations of the standard and the comparison stimuli is high. The TOE is also non-existent when the ISI becomes really long (longer than 4000 ss).

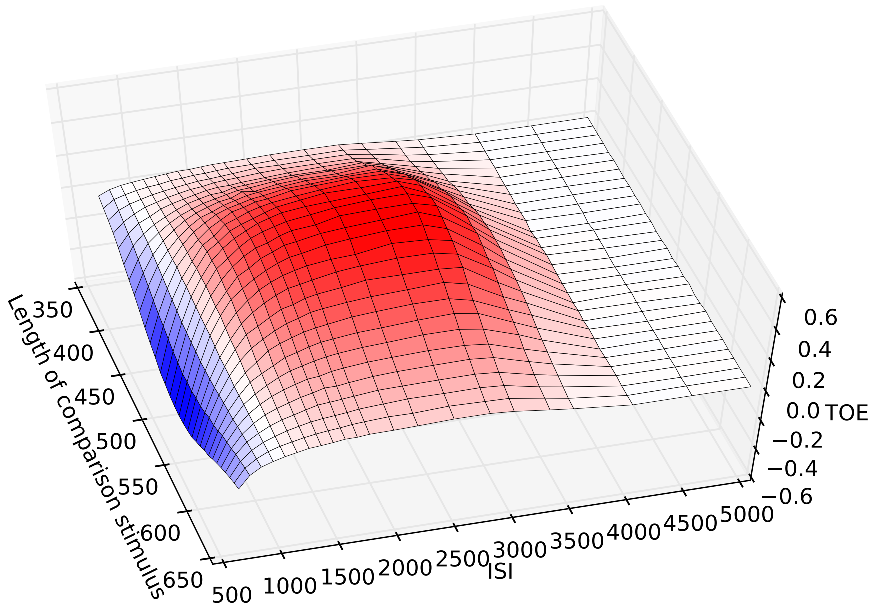

Figure 22: TOE values obtained in Experiment 3 for the standard stimulus lasting 500 ss. Accumulator Reset Rate (ARR) was high and Accumulator Bias Value (ABV) was low. Each point in this plot represents the TOE value calculated for a particular combination of durations of the comparison stimulus and the ISI.

3.2 Discussion

The initial analyses of the results described above indicate that Accumulator Reset Rate (ARR), Accumulator Bias Value (ABV), stimuli range and ISI strongly influence the magnitude and the polarity of the Type A, the Type B, the TOE effects and Weber fractions in the CCTN.

3.2.1 Type A

It is quite difficult to separate the influence of the ARR and the ABV on the Type A effect in the CCTN, as this effect results from an interplay between these two essential parameters. Regardless of the duration of the standard stimuli, when the ARR value is fixed, the higher value of the ABV increases the Type A effect (Fig. 11). This is because higher ABV values increase the preference towards the first stimulus in each presented pair. This, in turn, influences inversely the PSE points in the response functions for both presentation orders (Fig. 5). When the standard stimulus is presented first, the P(“C>S”) is generally decreased, as the standard is favored due to the higher ABV. Hence, the PSE for this presentation order is shifted in the direction of the longer comparison stimuli (a rightward horizontal shift). The opposite happens for the (C,S) response functions, resulting altogether in the increase of the Type A value. The increase of the ARR when the ABV is fixed results in a similar outcome, yet for a different reason. While the increase of the ABV entails a greater preference for the first stimulus, the increase of the ARR lowers the preference for the second one. Hence, the increase of these two parameter values yields the highest values of the Type A effect, which is confirmed by the regression analysis for Experiment 1 provided in Sect. 3.1.3, where the intensity of the Type A effect depends positively on both of these parameters.

The duration of the standard stimulus also influences the magnitude of the Type A effect. Generally speaking, the Type A value is more positive when the standard stimuli are shorter, and more negative when the stimuli last longer, which is especially visible in Experiment 2 (Fig. 13). This is not surprising, as when the standard stimuli durations are short, the ABV parameter is more influential than the ARR parameter. This causes the PSE (Fig. 6) for the (C,S) response function to be shifted in the direction of the shorter comparison stimuli (a leftward horizontal shift) and the PSE for the (S,C) response function to be shifted in the direction of the longer comparison stimuli (a rightward horizontal shift). In case of longer standard stimulus durations, the ARR parameter becomes more influential than the ABV parameter, yielding a leftward shift of the PSE values for the (S,C) response functions and a rightward shift of the PSE values for the (C,S) response functions, thus producing a negative Type A effect. This relation between ARR, ABV and the standard stimulus duration is also visible in Experiment 1 (Fig. 11) where ARR and ABV were not fixed and the durations of the standard stimulus increased from 300 ss to 700 ss – for each combination of ARR and ABV, the Type A effect decreased linearly (from a positive to a negative one) as the standard stimulus duration increased. The duration of the standard stimulus controls the influence of ARR (which determines $\text{TypeA}_{\text{slope}}$) and ABV (which determines $\text{TypeA}_{\text{intercept}}$) on the value of the Type A effect. For shorter standard stimuli, the ARR parameter is more influential than ABV, but when the standard stimulus becomes longer, the ABV parameter, relatively to the ARR parameter, gains more and more influence on the value of the Type A effect. Which standard stimulus is considered "shorter" or "longer" depends on the particular values of ARR and ABV, as determined by the coefficients of the regression equation provided in Sect. 3.1.3.

Finally, the ISI duration also influences the Type A effect. As the ISI gets longer, the CCTN mechanisms induce the change of the Type A value from negative to positive and back to zero (Fig. 15). This change is caused by the diminishing influence of the ARR parameter. A longer ISI means more time given to the CCTN resetting mechanism to clear the Accumulator, and this weakens the influence of the ARR and strengthens the role of the ABV. This happens until the ISI is long enough for the second stimulus in each pair to be presented in the same, base conditions as the first stimulus.

3.2.2 Type B

The Type B effect concerns the sensitivity characteristics of the comparison process of time stimuli. The values of the ARR and the ABV parameters influence the P("C>S") differently for either of the (C,S) and (S,C) orders. Because the comparison stimulus C varies across experimental trials, this influence is more complex for the (C,S) order of presentation than for the (S,C) order. Additionally, the P("C>S") characteristics is modulated by the stimuli range used in the experiment, which makes the trends concerning the Type B effect more complicated to explain than those for Type A.

When ARR is lower, the Type B effect is more negative (Fig. 12). This is because the resetting process of the Accumulator influences judgments differently in both presentation orders. In the (S,C) order, the remnant signal in the Accumulator is constant across the trials, while in the (C,S) order, longer comparison stimuli lead to a higher value of the remnant signal in the Accumulator. Hence higher values of P("C>S") are observed for the (S,C) order than for the (C,S) order across almost all of the comparison stimuli – a constant additional signal value is added to the comparison stimuli signal in the (S,C) order, and increasingly large additional value is added to the standard stimulus signal in the (C,S) order. Hence, the slope of the (C,S) response function is lower (yielding a larger DL value) than the slope of the (S,C) function. This effect is not universal and strongly depends on the duration of the standard and comparison stimuli, because the duration of these stimuli modulates the influence of the ARR parameter. The increase in ARR decreases the DL for the (C,S) order (Fig. 8). According to the assumptions of the CCTN, this is the case only if the lower value of the ARR parameter influences the DL of the (C,S) response function at all – i.e., when the lower value of ARR does not let the Accumulator to be entirely cleared. If the lower value of ARR leads to the total clearing the Accumulator, then the higher value of ARR does this as well, so increasing ARR would not introduce any difference in this situation. The observations provided in this paragraph demonstrate again that various experimental factors and internal parameters of the model strongly interact with each other to produce various timing response distortions, including changes of timing sensitivity.

The increase in the ABV parameter increases the Type B value. For the (C,S) order, this bias value added up to the signal in the Accumulator is higher during the presentation of the comparison stimulus than during the presentation of the standard stimulus (unless the ISI is sufficiently long). This difference in the added bias leads to the increase of the P("C>S") for this presentation order for each comparison stimulus. For the (S,C) order, by contrast, the pattern of bias addition is inverted. Taken together, the ABV-related mechanisms reduce the influence of ARR on DLs for both orders, thus increasing the Type B value.

The positive influence of the increase of ARR and ABV on the Type B values is confirmed by the regression equation determined for Experiment 1 in Sect. 3.1.3. This equation also indicates that the interaction between the parameters in yielding the Type B effect is more sophisticated than in the case of Type A – contrary to Type A, in Type B the ARR is relevant in both the slope and the intercept coefficients of the Type B characteristics.

Experiments 1 and 2 revealed that the Type B value decreases with the increase of the standard stimulus durations (Figs. 12 and 14). When the standard stimuli are shorter, the influence of the ARR change on the Type B phenomenon is less profound (the Accumulator is easier to clear regardless of the ARR value) compared to longer standard stimuli. The indirect evidence for this is that the slopes of the response functions for both presentation orders are steeper (which is indicated by lower DLs for these functions as shown in Figs. 8 and 9), and these slopes are also more similar to each other.

Not surprisingly, the ISI duration also influences the magnitude of the Type B effect (Fig. 16). Shorter durations of the ISI result in lower negative values of the Type B effect. This is because the ISI strongly interacts with the ARR parameter, influencing the time for the Accumulator to be cleared of the remnant signal after the presentation of the first stimulus. Longer durations of the ISI increase the Type B value, reducing the influence of the ARR parameter and strengthening the role of ABV up to the point where, for longer ISIs, the Type B effect becomes positive. For even longer ISIs, the Type B value starts to decrease to zero. Fluctuations of the Type B value indicate that this effect, given its range of values, is more sensitive to random changes of the stimulus signals in the network than the Type A effect. These fluctuations can take the form of local drops of the Type B values within a more global increasing trend. A decrease of the Type B value with increasing ISI has been also observed by [4]. Hence, in order to confront these predictions with the empirical data, real-life experiments should cover a wide range of experimental stimuli with many repeated trials.

3.2.3 Weber fraction

The analysis of the data concerning sensitivity ratios (WEB) yielded two interesting observations. First, the WEB trend for the averaged case does not trivially depend on the WEB trends for the two presentation orders. A more irregular situation is observed in case of a changing ISI (Fig. 19). These observations mean that WEB for averaged case is produced by a non-trivial interaction between the slopes of the (C,S) and (S,C) response functions. Second and even more interesting, the WEB values for the averaged case stabilize for shorter durations of the standard stimulus than the WEB values for the (C,S) and (S,C) orders (Fig. 18). This indicates that according to the CCTN, in a range of durations of the standard stimulus the Weber fraction values conform to Weber's Law, but only for the response functions averaged independently of the order of stimuli presentations, and not for the (S,C) and (C,S) response functions. As the exact WEB values obtained from the simulations presented in this work are only a bit lower than those reported in some studies [62, 72], adjusting the parameters of the CCTN will likely allow this model to fit the empirical human data, which is our ultimate goal.

3.2.4 The TOE

The TOE analysis reveals that the TOE is more negative when the values of both the ARR and the ABV parameters are low, and more positive when these values are high (Fig. 20). This is obviously because higher values of the parameters are more in favor of the overestimation of the first stimulus in each presented pair, and lower values result in the overestimation of the second stimulus in a pair. When the values of these two parameters are mixed (low-high or high-low), the observed patterns of changes of the TOE values are slightly more complicated, as the interplay between the parameters is especially sensitive to the context (the exact stimuli range, the ISI and possibly other factors, not included in this research, like the Pacemaker speed). The noticeable flattening of the trends in Figs. 20, 21 and 22 where TOE is nearly zero is an interesting emergent effect demonstrated by the CCTN. For the durations of the standard stimulus where the TOE is nearly flat, there is a nearly constant difference between the values of the (S,C) and (C,S) response functions across the entire tested range of the comparison stimulus. On the other hand, the Type B values are further from zero (*in plus*) for the shorter standard stimulus values corresponding to the *less* flat trends of the TOE. This means that the analysis of the TOE may provide extensive information about the comparison of the steepness of slopes of the (S,C) and (C,S) response functions, and, in turn, about timing sensitivity.

The range of standard stimuli also influences the TOE: more positive TOE values are obtained when the presented stimuli are short, and more negative TOE values result from longer stimuli (Fig. 21). This is not surprising, as longer stimuli signals are harder to be cleared from the Accumulator in a fixed time interval, hence increasing the preference for the second stimulus in a pair.

The asymmetry that is especially visible in the negative TOE series in Fig. 21 is caused by the fact that between the comparison stimuli C1 and C2 symmetrically selected around the standard stimulus ($\mathrm{S} - \mathrm{C}_1 = \mathrm{C}_2 - \mathrm{S}$ and $\mathrm{C}_1 \leq \mathrm{S} \leq \mathrm{C}_2$), the longer comparison stimulus (C_2) is more influential in yielding the negative TOE. For relatively long durations of the standard stimulus S, the P(CORRECT|FirstLonger) value is lower in the (S, C_1) case than in the (C_2, S) case, and the P(CORRECT|SecondLonger) value is higher in the (C_1, S) case than in the (S, C_2) case. The differences in the rates of correct answers are due to the Accumulator resetting process. A higher value is added to the second stimulus signal in the (C_2, S) case than in the (S, C_1) case, and a lower value is added to the second stimulus signal in the (C_1, S) case than in the (S, C_2) case.

The ARR and the ABV parameters clearly interact with the stimuli range – as, for example, the TOE lines cross when the standard stimuli get longer (Fig. 20) for the two mixed combinations of the parameter values (high-low and low-high). The ISI factor also influences the TOE, with an initial increase of the TOE values alongside the increase of the ISI durations, switching to the decrease and finally approaching zero (Fig. 22).

3.3 Summary

To sum up, the results of the simulations revealed that the CCTN is able to reproduce the Type A, the Type B and the TOE phenomena. Moreover, the results indicate that each of these phenomena can be influenced by a number of factors, but they all originate from a single timing mechanism. Interestingly, although one relatively simple timing mechanism induces all the three phenomena, the influence of the parameters of the CCTN and the external factors (such as stimuli durations or the ISI) on these phenomena can be quite complex. An example of such a non-trivial interplay is the covariability of the Type A and the Type B phenomena, which becomes difficult to analyze when the change of the standard stimulus duration results in a shift of the polarity of the Type A value. This causes profound changes in TOE values, at the same time keeping the magnitude of the Type B effect unchanged. It is of great advantage that such complex patterns concerning psychophysical quantities can be explained and predicted by simulations and quantitative analyses of the straightforward and transparent CCTN model; many of these patterns and relationships can be further verified in experiments on people and animals. One of the most interesting predictions is that the phenomenon similar to Weber's law may be caused and modulated by the same mechanism that causes the time-order error.

4 Conclusions

This paper discussed the Type A, the Type B and the TOE phenomena in the timing domain and explained how they emerge in the Clock-Counter Timing Network (CCTN) model. Simulation experiments revealed that two important mechanisms of the CCTN – the Accumulator resetting loop and the Accumulator bias maintenance system – induce all the three effects. The characteristics of these effects are, however, strongly dependent on the external factors such as stimuli duration and inter-stimulus interval, which is consistent with the literature [28, 2, 32, 4].

The CCTN model allows to perform experiments with parameter values changing dynamically during the course of experiments due to some cognitive mechanisms or external cues; the influence of such external cues on timing performance was discussed for example by [11]. This kind of influence of dynamically changing parameter values is one of the future tasks in the development of the model. The CCTN corresponds to other psychophysical timing models, like for example the Sensation-Weighting Model [30], in that it predicts that the role of stimuli in comparative judgments can differ and change depending on the context (e.g. relative stimuli durations, perceptual modality, etc.). However, the power of the CCTN model lies in the fact that it provides a functional explanation for this kind of effects and offers the interpretation of important psychophysical quantities (such as the PSE, the DL, and the Weber fraction) which refers to the information-processing mechanism.

The model and the presented methodology offer more than what has been included in this paper; some parameters of the CCTN that were not examined here can also strongly influence the Type A and B effects. Such parameters include the Pacemaker pulse rate and the interpulse interval distribution, which determine how much of the signal value is stored in the Accumulator during the presentation of stimuli [38] and, hence, may play a similar role to the Accumulator Reset Rate. On the other hand, the literature suggests that the Pacemaker speed depends on the modality in which stimuli are presented [71, 55, 63] and also on certain types of medicaments [52, 61, 10, 46]. For some technical parameters of the CCTN such as the Accumulator Bias Recovery Rate or the Accumulator–Working Memory transition speed, empirical operationalizations have yet to be developed.

To assess the Weber's Law property we used the difference limen (DL) as the indicator of the just-noticeable difference (JND). We have demonstrated that the CCTN model follows the property of Weber's law for longer durations of standard stimuli. The JND is not constant across the entire range of standard stimulus durations, and the point of its stabilization may depend on specific parameter values of the CCTN. The noticeable drop of Weber fraction values for longer durations of standard stimuli is in agreement with the literature [62, 63]. The examination of the CCTN reported in this paper suggests that the specific characteristics of the Weber's ratio functions depend on the parameters inducing the TOE. This indicates that according to the CCTN model, Weber's law and the TOE phenomena are interdependent.

Apart from the explanations summarized above and the predictions that can be verified in psychological experiments, the model and the methodology offers means for practical applications. The timing phenomena have been discovered to depend on drug application, diseases, or biological constitution in a number of medical experiments [37, 53, 68, 56, 70, 25, 65, 68, 64, 47, 23]. These distortions can be simulated in the CCTN to provide some diagnostic knowledge or to predict results of treatments. Yet another interesting issue would be to research and apply the knowledge about the influence of the Type A and B effects on the accuracy and the sensitivity of timing judgments. A sample application could concern efficient signalization systems in various environments that demand a high accuracy of human response, such as alarming and monitoring systems. These systems should be adjusted so that they provide feedback to humans taking into account the knowledge about human time perception. Based on the model of an imperfect human internal perception mechanism, such systems could dynamically compensate for (or take advantage of) the discovered human biases to produce anti-biased (i.e., deliberately inaccurate) signals so that they are recognized or compared by a human with expected accuracy.

Finally, future research also includes the examination of the Type A, the Type B and the TOE effects in more complex multi-agent environments and in simulated evolutionary processes supported by existing software [43]. In evolution, the parameters of the CCTN and its structure may change while agents equipped with a working CCTN are selected based on their fitness in a given environment. This would enable tests of the adaptive value of the TOE and related phenomena and the identification of the environmental characteristics that make the TOE-biased agents to be advantageous or disadvantageous compared to unbiased agents. Such research will be an advancement towards a holistic description and explanation of these effects that are as ubiquitous in human perception as the timing itself.

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